IB MYP YEAR 5

ASSESSMENT TASK

A Broad-based Test

SOLUTIONS

Subject:	Y10 ***Extended*** Mathematics	**Name : (Class)**	( )
Assessment:	**Broad-based Test**		
Topics covered	**Polynomials, Transformations (and transforming functions), Probability, Vectors, Matrices, Indices**		
Date of assessment:	**Thursday 16th February 2012**		

- **This task assesses Criteria A and C;**
- **Time allowed – *one hour 40 minutes;***
- **You must answer <u>all</u> the questions;**
- **Write your answers in the spaces provided;**
- **Show all of your working – <u>not</u> just the answer**
- **GDCs are allowed.**

Criterion A

Levels	Task-Specific Rubric	Official IB Descriptors
0	The student does not reach a standard described by any of the descriptors given below.	
1-2	Students are reasonably successful with the Part A questions only. Any errors here are relatively minor.	The student **generally** makes appropriate deductions when solving **simple** problems in **familiar** contexts.
3-4	Students are successful with Part A questions. The only errors in part B questions are minor.	The student generally makes appropriate deductions when solving **more complex** problems in **familiar** contexts.
5-6	Students are successful with Part A and B questions. The only errors in part C questions are minor.	The student **generally** makes appropriate deductions when solving **challenging** problems in a **variety** of **familiar** contexts.
7-8	Students are successful with Part A, B and C questions. The only errors in part D questions are minor.	The student **consistently** makes appropriate deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar situations**.

Criterion C

Levels	Task-Specific Rubric	Official IB Descriptors
0	The student does not reach a standard described by any of the descriptors given below.	
1-2	Very little working is shown, and/or the steps shown are confusing. Only the most basic mathematical symbols are used with accuracy.	The student shows **basic use** of mathematical language and/or forms of mathematical representation. The lines of reasoning are **difficult to follow.**
3-4	The working shown is generally adequate. Only a few errors in symbols/terminology are evident. It is reasonably easy to follow a student's logic/reasoning.	The student shows **sufficient use** of mathematical language and forms of mathematical representation. The lines of reasoning are **clear though not always logical or complete.** The student moves between different forms of representation with **some success.**
5-6	There are very few, if any, errors in symbols/terminology. All steps in calculations are shown in their completeness. It is easy to follow all the student's logic/reasoning.	The student shows **good** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **concise, logical** and **complete.** The student moves **effectively** between different forms of representation.

Part A (Level 1-2 Questions)

Q1. Factorise ***completely*** the following expressions:

(a) $x^2 - 16$

$= x^2 - 4^2$

difference of 2 squares

Answer (a) $(x-4)(x+4)$

(b) $x^2 - 16x$

$= x(x-16)$

Answer (b) $x(x-16)$

(c) $x^2 - 16x - 36$

$= (x-18)(x+2)$

Answer (c) $(x-18)(x+2)$

Q2. Simplify the following expressions, giving your answers in the form a^n, where a and n are integers:

(a) $3^5 \times 3^{-4}$

$= 3^{5+(-4)}$

Answer (a) 3^1

(b) $2^4 \times 3^4$

$= (2 \times 3)^4$

Answer (b) 6^4

(c) $(6^2)^3$

$= 6^{2\times 3}$

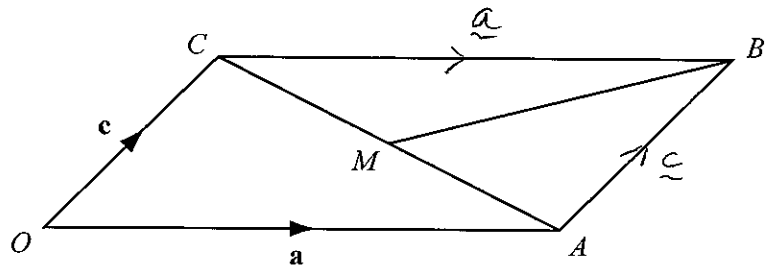
Answer (c) 6^6

(d) $16^2 \div 2^8$

$= \frac{(2^4)^2}{2^8} = \frac{2^8}{2^8}$

Answer (d) 2^0

Q3.



OABC is a parallelogram. $\overrightarrow{OA}$ = **a**, $\overrightarrow{OC}$ = **c** and M is the midpoint of CA. Find, in terms of **a** and **c**:

(a) $\overrightarrow{OB}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \underset{\sim}{a} + \underset{\sim}{c}$

Answer (a) $\underset{\sim}{a} + \underset{\sim}{c}$

(b) $\overrightarrow{CA}$

$\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA}$

$= -\underset{\sim}{c} + \underset{\sim}{a}$

Answer (b) $\underset{\sim}{a} - \underset{\sim}{c}$

(c) $\overrightarrow{BM}$

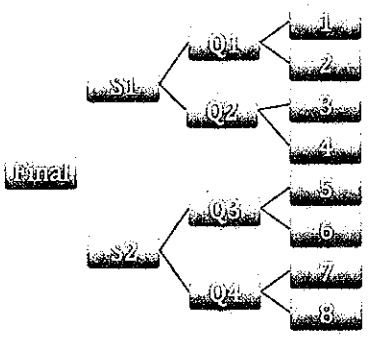
$\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BO}$ (diagonals of a parallelogram bisect each other)

$= -\frac{1}{2}\overrightarrow{OB}$

Answer (c) $-\frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{c})$

Part B (Level 3-4 Questions)

Q4. Eight teams take part in a basketball tournament. Each team is equally likely to win any particular game. Winning teams advance to the next stage. The losers go home!!



* This is a misleading question - all 8 teams are already in the Q-Finals !!

accept as "win"

(a) Find the probability that all the even numbered team reach the quarter-finals (Q1, Q2, Q3, Q4).

$P = (2 \text{ beats } 1 \text{ AND } 4 \text{ beats } 3 \text{ AND } 6 \text{ beats } 5 \text{ AND } 8 \text{ beats } 7)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$ (independent events)

NB (see above *)
Logically we have to accept an answer of 1

Answer (a) 1/16

(b) Find the probability that team 1 will play team 8 in the final.

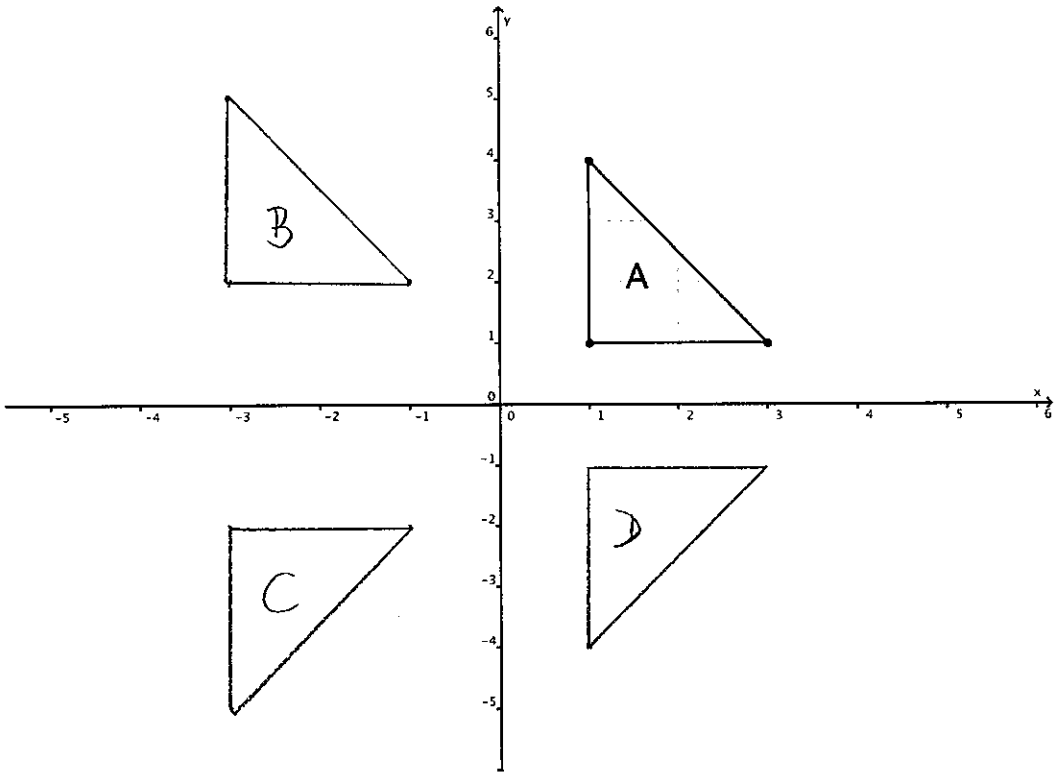
$P(1 \text{ reaches final}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(8 \ldots \ldots) = 1/4$ also

$\Rightarrow P(\text{both reach final}) = \frac{1}{4} \times \frac{1}{4}$ (independent events)

Answer (b) 1/16

Q5. Triangle A is plotted on the Cartesian plane below.



(a) On the same Cartesian plane above, draw the following:

(i) Triangle A after it is translated by a translation vector $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$, label it B.

(ii) Triangle C is the reflection of triangle B in the *x*-axis, draw triangle C.

(iii) Triangle D is the reflection of triangle A in the *x*-axis, draw triangle D.

(b) Describe the single transformation, which maps triangle C onto triangle D.

Translation, 4 units right, 1 up

Answer $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

Q6. Erik runs a race at an average speed of x m/s.
His time is $(3x - 9)$ seconds and the race distance is $(2x^2 - 8)$ metres.

(a) Write down an equation in x and show that it simplifies to

$x^2 - 9x + 8 = 0$

average speed $= \frac{\text{distance}}{\text{time}}$

$\Rightarrow \quad x = \frac{2x^2 - 8}{3x - 9} \quad \Rightarrow \quad x(3x - 9) = 2x^2 - 8$

$\Rightarrow 3x^2 - 9x = 2x^2 - 8$

$\Rightarrow x^2 - 9x + 8 = 0$ Q.E.D

(b) Solve $x^2 - 9x + 8 = 0$

$\Rightarrow (x - 8)(x - 1) = 0$

$\Rightarrow x - 8 = 0$

or $x - 1 = 0$

Answer (b) $x = 1$ or $x = 8$

(c) Write down Erik's time and the race distance.

Of the two solutions, we can ignore $x = 1$ since this gives a negative distance and time

$\Rightarrow x = 8 \quad \Rightarrow$ Time $= 3 \times 8 - 9 = 15$ s

and Distance $= 2(8)^2 - 8 = 120$ m

Answer (c) 15 s, 120 m

Part C (Level 5-6 Questions)

NB if students the 0.6% was an ANNUAL rate, accept appropriate working and answer

Q7. Luis deposits a large sum of money in a bank account that pays 0.6% interest, compounded monthly. How long does it take Luis's money to grow by 10%?

After n months, Luis' money is worth $(1.006)^n X$

(where X is the principal)

We require:

$(1.006)^n X = 1.1X$

ie $(1.006)^n = 1.1$

$\Rightarrow n = 15.93$

May see Trial + error
May see solution via logs
May " graphical solution
" " GDC G-solve solution

Answer 16 months

Q8. A is the matrix $\begin{pmatrix} 5 & 2 \\ 2 & 0 \end{pmatrix}$ and AB is the matrix $\begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$. Find the matrix B.
(Remember the order of matrix multiplication matters!)

$AB = X$

$\Rightarrow A^{-1}AB = A^{-1}X$

$\Rightarrow B = A^{-1}X$

$= \frac{-1}{4}\begin{pmatrix} 0 & -2 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$

$= \frac{-1}{4}\begin{pmatrix} -88 & -16 \\ 198 & 36 \end{pmatrix}$

$= \begin{pmatrix} 22 & 4 \\ -\frac{99}{2} & -9 \end{pmatrix}$

Answer $B = \begin{pmatrix} 22 & 4 \\ -49.5 & -9 \end{pmatrix}$

Q9. Solve the following equation for x:

$4^{x+1} + 4^x + 4^{x-1} = 42$

Let $4^{x-1} = y \Rightarrow 16y + 4y + y = 42$

$\Rightarrow 21y = 42$

$\Rightarrow y = 2 \Rightarrow 4^{x-1} = 2$

$\Rightarrow (2^2)^{x-1} = 2^1 \Rightarrow 2^{2x-2} = 2^1$

$\Rightarrow 2x - 2 = 1 \Rightarrow x = 3/2$

Answer $x = 1.5$

Part D (Level 7-8 Questions)

Q10. You are given the quadratic equation $x^2 + kx + 4 = 0$

(a) What values of k ensure that this equation has repeated real roots?

Repeated roots $\Rightarrow \Delta = 0 \Rightarrow k^2 - 16 = 0$

$\Rightarrow k = \pm 4$

Answer (a) $k = \pm 4$

(b) It is decided that k is an integer. A student chooses a value of k at random from the interval

$-5 \leq k \leq 5$

What is the probability that the resulting quadratic equation has no real roots?

No real roots $\Rightarrow b^2 - 4ac < 0$

$\Rightarrow k^2 - 16 < 0$

$\Rightarrow k^2 < 16 \Rightarrow -4 < k < 4$

i.e. if $k \in \{-3, -2, -1, 0, 1, 2, 3\}$ there are no real roots

The interval $-5 \leq k \leq 5$ contains 11 integers

$\Rightarrow p(\text{no real roots}) = 7/11$

Answer (b) .....

(c) You are told to transform the original quadratic to $y = x^2$ **in one step**. What value of k would you choose, and what transformation would you perform?

If $k = 0$, original equation is $y = x^2 + 4$

This can be transformed to $y = x^2$ via the $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ translation

If $k = 4$, original equation is $y = x^2 + 4x + 4 = (x+2)^2$

This can be transformed to $y = x^2$ via the $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ translation

If $k = -4$, we have $y = (x-2)^2$

which can be transformed to $y = x^2$ via the $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ translation

Answer (c) .....

Q11. $A = \begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} x \\ y \end{pmatrix}$

We are told that AB = kB (where k is an integer).

(a) Using AB = kB, set up and simplify two simultaneous equations with x and y in.

$AB = kB \Rightarrow \begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = k\begin{pmatrix} x \\ y \end{pmatrix}$

$\Rightarrow \quad x + 6y = kx$

$4x + 3y = ky$

$\Rightarrow \quad 6y = (k-1)x \quad -①$

and $4x = (k-3)y \quad -②$ or similar

(b) By solving the simultaneous equations in (a), find possible values for k.

① gives $x = \frac{6y}{k-1}$

Sub in ②: $\frac{24y}{k-1} = (k-3)y$

$\Rightarrow 24 = (k-3)(k-1)$

$\Rightarrow k^2 - 4k - 21 = 0$

$\Rightarrow (k-7)(k+3) = 0$

$\Rightarrow k = -3$ or 7

Answer $k = -3$ or $k = 7$

NOW GO BACK AND CHECK YOUR WORK